Velocity Control of Electric Propulsion Space Vehicles Using Heliocentric Gravitational Sling

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Abstract - The objective of this research is to determine a near optimal fuel efficient control strategy for an electric propulsion space vehicle to attain a predefined velocity at the target. The control strategy is represented as a chromosome and micro genetic algorithms are used to find the best solution. The control strategy uses the space vehicle’s reverse thrust and the sling effect caused by the Sun.

INTRODUCTION
The motivation behind this work is to traverse from Earth to the moons of Jupiter in Jupiter’s Icy Moon Orbiter (JIMO). JIMO is scheduled to be launched in 2015, and it is projected that by then the technology will be available for the prolonged usage of ionic propulsion. Numerous technological advancements in high-precision sensors, autonomous control, and automation have made it possible to go beyond previous expectations. Deep Space 1, launched on October 24, 1998 from Cape Canaveral, is a case in point. On its course to comet Borrelly, many technological advancements, such as solar electric propulsion (SEP), solar concentrator arrays, autonomous navigation, and the small deep space transponder, were successfully tested [Rayman, M.D., Varghese, P., Lehman, D. H. and Livesay. L.L, 1999]. Even though Deep Space 1 used the ion propulsion system (IPS) for only brief periods, it has validated its usage for future missions.

There are two major research areas in the field of interplanetary space science missions: spacecraft design and mission planning. The focus of this research is on unconventional methodologies to achieve near optimal trajectory planning in a timely manner. Since transporting fuel to space is expensive, the objective is to schedule the engine thrust operations (interval and direction of the thrust) during the mission so that a nearly minimal amount of fuel is used to carry out the mission successfully.

NASA is in a quest for new technologies for deep space exploration. These missions cannot be achieved with solid or chemical propulsion engines. Such engines will be used to transfer the spacecraft from the Earth’s surface to a low Earth orbit (LEO). Thereafter, low-thrust electric propulsion may be used to re-orbit and to spiral JIMO gradually away from the Earth’s gravity and to station it in orbit about another planet (for example, Jupiter’s). The multi-body effect of nearby planets makes this voyage more complicated. The effects are particularly intense near escape and capture.

The ability to regulate the velocity of JIMO at the destination is a paramount factor in determining capture by the target planet and subsequent navigation. High velocities could cause JIMO to burn up while entering the atmosphere of the target planet. The control profiles of JIMO are thrust on (thrusting) versus thrust off (coasting), the thrust angle of attack and the roll angle (or bank angle), and its states are defined by its mass, longitude, latitude, radius, heading angle, flight path angle and the velocity. The time of launch and target intercept are calculated based on various factors such as the proximity of the target coordinates from the Earth and the positions of intermediate celestial bodies.

An optimal control strategy to minimize fuel consumption using discrete thrusting (thrust on and off) is addressed in [Kirk, D.E., 1970, pp 260-284]. This method is commonly referred to as Bang-Bang control. It becomes extremely tedious and impractical to implement because of highly nonlinear and coupled dynamic equations defining JIMO’s position relative to any celestial body.

Trajectory design tools that patch together low-thrust arcs of predefined trajectory shapes were developed by [Petropoulos et al] and [Petropoulos and Longuski]. Trajectories were also obtained by approximating these low-thrust arcs by a series of impulsive thrusts [Sims and Flanagan], [McConaghy et al]. These tools were used in [Debban et al] to develop low-thrust gravity assist trajectories for flight to Mercury, Pluto and Jupiter. This work also makes a good comparison of trajectories obtained using different tools. Most of the similar work focuses on optimizing predefined candidate trajectories or patching together individual thrust arcs.

Stochastic processes, such as simulated annealing, could also be used for optimization of trajectories, as demonstrated by [Tekinalp and Bingol]. That work focused on optimizing a missile trajectory, which is ballistic in nature.

Genetic algorithms are used in a wide range of optimization problems [L. Chambers]. They were used in optimizing the fuel consumed by solar-electric propulsion systems [Williams and Coverstone-Carroll, 2000] for Earth-Mars and Mars-Earth trajectories. Representation of the
thrust sequence by [Coverstone-Carroll,V., 1996] is of particular interest for this research. By using genetic algorithms (GAs), the overall complexity of the problem is drastically reduced. Another advantage of using a GA over other heuristic methods is the relative simplicity of the GA’s chromosome representation in modeling the overall control strategy for this mission. The fuel efficiency and JIMO’s trajectories are analyzed for increasing mission durations.

**PROBLEM DEFINITION**

The objective of this research is to develop a near optimal fuel-efficient control strategy enabling JIMO to reach its target coordinates with a predefined velocity. The control profiles of the spacecraft are thrust on (thrusting and braking) versus thrust off (coasting), the thrust angle of attack and the roll angle of the spacecraft. The trajectory being investigated is for the heliocentric case.

**SPACECRAFT DYNAMICS**

The spacecraft is modeled around a point mass representation in space [Vinh, N.X, 1981]. The equations of its motion are as follows.

\[
\begin{align*}
\frac{dr}{dt} &= V \sin \gamma \\
\frac{d\theta}{dt} &= \frac{V \cos \gamma \cos \psi}{r \cos \phi} \\
\frac{d\phi}{dt} &= \frac{V \cos \gamma \sin \psi}{r} \\
\frac{dV}{dt} &= -g \sin \gamma + \omega^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \sin \psi) \\
\frac{d\gamma}{dt} &= \frac{F_s \cos \sigma}{mV} - \frac{g \cos \gamma}{V} + \frac{V \cos \gamma}{r} + 2 \omega \cos \phi \cos \psi \cos \phi \sin \psi \\
&\quad + \frac{\omega^2 r}{V} \cos \phi (\cos \gamma \cos \phi + \sin \gamma \sin \phi \sin \psi) \\
\frac{d\psi}{dt} &= \frac{F_s \sin \sigma}{mV \cos \gamma} - \frac{V}{r} \cos \gamma \cos \phi \tan \phi \\
&\quad + 2 \omega (\tan \gamma \cos \phi \sin \psi - \sin \phi) - \frac{\omega^2 r}{V \cos \gamma} \sin \phi \cos \phi \cos \psi \\
\frac{dm}{dt} &= -\frac{T}{g_s I_{sp}}
\end{align*}
\]

The spacecraft’s thrust magnitude is 3.787 N when the engines are firing, and the mass consumption of the engines is 6.787e-5 Kg/sec. The control parameters of the spacecraft are its thrust (T), the thrust angle of attack (α) and the roll angle (σ). These parameters are as illustrated in Figure 2. The angle of attack is the angle between the thrust and the velocity vector. The drag and lift created on the spacecraft are represented by \( \vec{D} \) and \( \vec{L} \), respectively. The roll angle is the displacement of the lift vector from the local vertical plane of the spacecraft. The thrust of the spacecraft has two components, a tangential component and a vertical component, defined by

\[
\begin{align*}
F_T &= T \cos \alpha - D \\
F_N &= T \sin \alpha + L
\end{align*}
\]

Finally, the gravitational acceleration is given by \( g = \mu / r^2 \)

where \( \mu = 1.327 \times 10^9 \).
Figure 2- Spacecraft Control Parameters

GENETIC ALGORITHMS

Genetic algorithms [Holland, J., 1992 and Goldberg, D.E., 1989] use a heuristic search process inspired by the mechanics of natural selection. To obtain JIMO’s control strategy, micro GAs with elitism are selected over Simple GAs and Simple Gas with elitism due to their simplicity and computational efficiency [Reddy et all]. This form of GA is illustrated in Figure 3. A population of \( n \) chromosomes is randomly chosen to create an initial pool of candidate solutions, and their fitness is calculated based on equation 1. Tournament selection is used to select two dissimilar chromosomes for crossover. Let NC1 and NC3 be the resulting children, with NF1 and NF3 their corresponding fitness values. NC1 is checked against all the chromosomes in the initial population. If an exact match is not found, and if NF1 is greater than T1, then the chromosome corresponding to T1 is replaced by NC1 and T1 by NF1. A search for the new minimum fitness in the population is made, and the same process is repeated for NC3. A randomly chosen chromosome is mutated and accepted only if it enhances the overall population. These stages are repeated until a solution is obtained.

A chromosome for the JIMO’s control strategy is a sequence of segments where each segment simulates a duration of two days. Each segment consists of two thrust bits, several bits for the thrust angle of attack, and several bits for the roll angle of the spacecraft, in that order. An individual chromosome is modeled as

\[
\text{Chromosome} = (\text{Segment 1}) + (\text{Segment 2}) + \ldots + (\text{Segment n}).
\]

Each segment represents the thrust magnitude, thrust angle of attack and roll angle of the spacecraft in the following order.

Individual Segment = Thrust Bits (Two Bits) + Thrust Angle of Attack (Multiple Bits) + Roll Angle of Spacecraft (Multiple Bits).

For thrust combinations, bits 00 indicate braking, bits 11 indicate thrusting, and bits 10 and 01 both indicate coasting. The choice of the number of bits used to represent a particular control variable in the chromosome is based on the desired level of accuracy. The accuracy and the number of bits used are given in Table 1. The number of bits used to represent each variable is given by

\[
\text{Accuracy} = \frac{\text{Upper Bound} - \text{Lower Bound}}{2^n - 1}
\]

Table 1. Accuracy and range of the control parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Accuracy</th>
<th>Lower Bound</th>
<th>Upper bound</th>
<th>Bits used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward Thrust Angle</td>
<td>2 Deg</td>
<td>-45 Deg</td>
<td>45 Deg</td>
<td>6</td>
</tr>
<tr>
<td>Reverse Thrust Angle</td>
<td>0.2 Deg</td>
<td>175 Deg</td>
<td>185 Deg</td>
<td>6</td>
</tr>
<tr>
<td>Roll Angle of Spacecraft</td>
<td>3 Deg</td>
<td>-180 Deg</td>
<td>180 Deg</td>
<td>7</td>
</tr>
</tbody>
</table>

The fitness function for this application is given by equation 1. This equation demonstrated its general versatility for most of the target-seeking problems.

\[
\text{Fitness} = \frac{M}{(v^2 + e_1^2 + e_2^2 + e_3^2 + e)}
\]

where,

\[
M = \frac{\text{Fuel at pt of Intercept}}{\text{Initial Mass of Fuel}}
\]

\[
e_1 = \frac{\text{Desired Radius - Radius at pt of Intercept}}{\text{(Desired Radius - Initial Radius)}} \times 100
\]

\[
e_2 = \frac{\text{Desired Longitude - Longitude at pt of Intercept}}{\text{(Desired Longitude - Initial Longitude)}} \times 100
\]

\[
e_3 = \frac{\text{Desired Latitude - Latitude at pt of Intercept}}{\text{(Desired Latitude - Initial Latitude)}} \times 100
\]

\[
v = \frac{\text{Desired Velocity - Actual Velocity at the Point of Intercept}}{1000}
\]

The very small value \( \in \) is added to the denominator to guard against the unlikely case where \( e_1, e_2, \) and \( e_3 \) are all zero, which (without adding \( \in \) ) would result in division by zero. The objective is to maximize the fitness function. \( M \) is the percent of fuel remaining at the end of the flight; this value should be as large as possible. \( e_1 \) is the error in the
final radial coordinate as a percentage of the change in this coordinate from the start to the end of the flight. Likewise, \( e_2 \) is the error in the final longitudinal coordinate as a percentage of the change in this coordinate, and \( e_3 \) is the error in the final latitudinal coordinate as a percentage of the change in this coordinate. Values \( e_1, e_2, \) and \( e_3 \) measure error and they, as well as their sum, should be as small as possible. Thus, the multiplicative inverse of their sum should be as large as possible. In the fitness function, the inverse of the sum of the squares of the error terms occurs since then the GA is more tolerant of small errors (and so is not driven to overly compensate) and less tolerant of large errors (and so does not wander too far from the goo solutions).

By formulating the fitness function as Equation 1, the dimensional units of radius, longitude, latitude and mass are all brought to a common scale. Restricting all quantities to a common dimensionless range eliminates the competition of various factors and presents the function value as a unitary figure of merit. Significant improvement in this figure can be obtained only by simultaneous improvement in fuel efficiency and flight precision. Near optimal performance is obtained using this fitness function as discussed in the next section.

Since almost all final velocities in trajectory simulations were in the range of a few thousand to a few tens of thousand kilometers per second, dividing the absolute value of the error in the final velocity by 1000 (as done here to calculate \( v \)) gives an error measure in a range consistent with that of the other error terms. The velocity is regulated using this fitness function.

**SIMULATION RESULTS**

A micro-population of 20 chromosomes was used as the initial GA population. The probability of mutation was 0.02 percent. The spacecraft is assumed to be in geocentric lower orbit with an initial mass of 500 kg, velocity of 15 km/sec, flight path angle of 45 degrees, and a heading angle of 40 degrees. Through a process of trial and error, the minimum duration required to complete this mission was determined to be 700 days (350 segments, each segment simulating a period of two days). The trajectory of JIMO using the control strategies determined for the mission (vide infra) is shown in Figure 4. Here JIMO exploits the Sun's gravitational force to achieve a sling effect. In this figure, the Sun is at the origin of the coordinate system, and the figure extends into the negative portions of all three axes. Since the perspective is down on the Sun at an angle, the Sun appears projected away from the origin in the grid on the \( Z = -1 \) plane. The control strategies determined for this mission are given in Figures 5 and 6.
Figure 7 shows JIMO’s distance from the Sun throughout the mission. There is a cusp at segment 220 where JIMO is closest to the Sun and switches from braking to thrusting as it flies past.

Figure 7 – Change in JIMO’s Radius

Figure 8 shows JIMO’s longitude as the mission unfolds. Throughout the mission, JIMO’s motion is always in the direction of increasing longitude in this left-handed coordinate system. Around segment 220, the longitude increases rapidly then shows an apparent discontinuity when it reaches $2\pi$ and jumps back to 0. In fact, this is an artifact of the representation since $2\pi$ is equivalent to 0, and longitude is restricted to the range $0 - 2\pi$.

Figure 8 – Change in JIMO’s Longitude

JIMO’s latitude throughout the mission is shown in Figure 9. It starts at a modestly positive latitude, which increases slightly until JIMO nears the Sun, when the latitude decreases, eventually drastically, nearly reaching the $Z = 0$ plane (on which the Sun lies). As JIMO moves away, the latitude increases drastically and stabilizes to the target latitude well before the target is reached.

Figure 9 – Change in JIMO’s Latitude

Figure 10 shows the fuel remaining as the mission progresses. Fuel is consumed at a nearly constant rate of about 4.8 kg/day.

Figure 10 – JIMO’s Fuel Consumption

The statistics for the fitness values with respect to the iteration count are shown in Figure 11. The final state of the JIMO spacecraft using this control strategy is given in Table 2.
Thus the duration cannot be linearly mapped to fuel efficiency. Maximizing fuel efficiency in an electric propulsion system is a complex problem intertwined with other factors. Nevertheless, finding a near optimal tradeoff between time and fuel efficiency is important for deep space missions. This work can be extended by using hybrid forms of evolutionary algorithms, such as simulated annealing with GAs. Also, the dynamics of JIMO’s flight, analyzed here as a two-body problem, can be extended to include multiple bodies to provide more realistic control strategies.

REFERENCES


**NOMECLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$\omega$</td>
<td>Angular rate of Earth’s rotation</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Gravitational constant</td>
</tr>
<tr>
<td>$g_r$</td>
<td>Gravitational acceleration at Earth’s Surface</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td>Specific impulse of spacecraft</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thrust angle of attack.</td>
</tr>
<tr>
<td>$T$</td>
<td>Thrust</td>
</tr>
<tr>
<td>$r$</td>
<td>Distance between celestial object and spacecraft</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Longitude</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Latitude</td>
</tr>
<tr>
<td>$V$</td>
<td>Speed of spacecraft</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Flight path angle</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Heading Angle</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Bank Angle or Roll Angle.</td>
</tr>
<tr>
<td>$m$</td>
<td>Fuel mass of the spacecraft</td>
</tr>
</tbody>
</table>