ABSTRACT
This paper presents an application of model-based residual generation for fault detection and isolation (FDI) to an induction machine. A scheme of observers with nonlinear decoupling is used for residual generation. The resulting residuals are robust to load torque disturbances and to parameter uncertainties such as moment of inertia and friction while being sensitive to stator and rotor electrical faults.

KEYWORDS: Fault Diagnosis, Residual generation, Nonlinear system, Observer, Robustness, Induction machine.

1. INTRODUCTION
Fault diagnosis of induction motors has received recently a great deal of attention [1], [4], [5], [6]. The nonlinear dynamics of the induction motor makes FDI a difficult task especially for multiplicative parametric faults such as the stator and rotor electrical faults. Different FDI methods have been applied to the induction machine ranging from state and parameter estimation to signal processing techniques.

The model-based FDI approach, also known as the analytical redundancy approach, basically consists in checking if the actual system behaviour is consistent with its mathematical model. It involves two main steps: residual generation and decision making. Residuals which are computed on-line from the measured inputs and outputs of the system should be useful fault indicators: they should be small in normal operation and should deviate from zero when particular faults occur. These residuals are then evaluated in order to draw a diagnostic decision (detection, isolation) which should be reliable (minimum rate of missed detections and false alarms). The central issue of robust residual generation in the face of model uncertainties and external disturbances has been extensively investigated for linear systems [2], [3], [7], [8], [11] and to a lesser extent for nonlinear dynamic systems [9], [10], [12], [13], [14].

In this paper we focus on the robust residual generation problem based on the nonlinear model of an induction motor (IM) which is basically an unknown input decoupling problem. It is achieved as a scheme of unknown input observers with nonlinear decoupling. This observer scheme aims to generate structured residuals which enable detection and isolation of stator and rotor electrical faults in the presence of mechanical parameter uncertainties and load disturbances. The paper is organized as follows. The diagnostic model of the induction model is presented in section 2, starting from the classical Park model. In section 3, the robust residual generation scheme is presented and the robustness and sensitivity issues are discussed. In section 4 simulation results are reported to evaluate the performance of the FDI scheme.

2. DIAGNOSTIC MODEL OF THE INDUCTION MOTOR
Assuming linear magnetic circuits and a balanced three-phase system in the reference frame \((a, b, c)\), the electrical equations of the induction motor expressed in the two-phase reference frame \((d, q)\) are:
\[
\begin{bmatrix}
\Phi_{sd} \\
\Phi_{sq} \\
\Phi_{rd} \\
\Phi_{rq}
\end{bmatrix} =
\begin{bmatrix}
L_s & 0 & L_m & 0 \\
0 & L_s & 0 & L_m \\
L_m & 0 & L_r & 0 \\
0 & L_m & 0 & L_r
\end{bmatrix}
\begin{bmatrix}
I_{sd} \\
I_{sq} \\
I_{rd} \\
I_{rq}
\end{bmatrix}
\]  
\[(1)\]

\[
V_{sd} = R_s I_{sd} + \dot{\Phi}_{sd} - \Phi_{sq} \dot{\theta}_s \\
V_{sq} = R_s I_{sq} + \dot{\Phi}_{sq} + \Phi_{sd} \dot{\theta}_s \\
V_{rd} = 0 = R_r I_{rd} + \dot{\Phi}_{rd} - \Phi_{rq} \dot{\theta}_r \\
V_{rq} = 0 = R_r I_{rq} + \dot{\Phi}_{rq} + \Phi_{rd} \dot{\theta}_r
\]  
\[(2)\]

where \( \Phi, I, V \) are the stator/rotor fluxes, currents and voltages expressed in the \((d,q)\) reference frame. \( \theta_s \) is the angle between the stator reference frames \((a,b,c)\) and \((d,q)\), and \( \theta_r \) is the angle between the rotor reference frames \((a,b,c)\) and \((d,q)\). \( R_s, R_r, L_s, L_r \) are the stator/rotor resistances and inductances, and \( L_m \) is the magnetizing inductance. For a squirrel-cage IM the rotor voltages are zero. The mechanical equation is:

\[
J \ddot{\Omega} = T_e - f_c \Omega - T_L
\]  
\[(3)\]

and the electromagnetic torque \( T_e \) is given by:

\[
T_e = pL_m (I_{sq} I_{rd} - I_{sd} I_{rq})
\]  
\[(4)\]

\( \Omega, J, f_c \) are the rotor speed, inertia and friction, \( p \) is the number of pole pairs and \( T_L \) is the load torque. Using the Park transformation with the reference frame \((d,q)\) fixed to the stator (i.e with \( \theta_s = 0 \) and \( \dot{\theta}_s = -p\Omega \)), Eq. (1),(2),(3) and (4) are transformed to the following nonlinear state space equations:

\[
\dot{x} = A(x) + Bu \\
y = Cx
\]  
\[(5)\]

where the state, input and output vectors are defined as:

\[
x = \begin{bmatrix} I_{sd} & I_{sq} & I_{rd} & I_{rq} & \Omega \end{bmatrix}^T, \quad u = \begin{bmatrix} V_{sd} & V_{sq} \end{bmatrix}^T, \quad y = \begin{bmatrix} I_{sd} & I_{sq} \end{bmatrix}^T
\]

\[
A(x) = \begin{bmatrix}
\xi (-L_s R_s x_1 + L_m^2 px_3 x_2 + L_m R_s x_3 + pL_r L_m x_5 x_4) \\
\xi (-L_m^2 px_5 x_1 - L_s R_s x_2 - pL_r L_m x_5 x_3 + L_m R_s x_4) \\
\xi (L_m R_s x_1 - pL_s L_m x_5 x_2 - L_s R_s x_3 - L_s L_r px_5 x_4) \\
\xi (pL_m L_m x_5 x_1 + L_m R_s x_2 + L_s L_r px_5 x_3 - L_s R_s x_4) \\
\frac{1}{J} (pL_m L_m x_5 x_3 - pL_m L_m x_5 x_4 - f_c x_5 - T_L)
\end{bmatrix}
\]

\[
B = \begin{bmatrix} \xi L_r \\
-\xi L_m \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}, \quad \xi = 1/(L_s L_r - L_m^2)
\]

The stator currents \( I_{sd}, I_{sq} \) and the rotor speed \( \Omega \) are assumed to be measured since they are usually used for control purposes. As a matter of fact, measurements of stator currents and voltages are made in the \((a,b,c)\) reference frame but they can be expressed in the \((d,q)\) reference frame and vice-versa, thanks to the Park transformation:
\[
\begin{bmatrix}
V_{sd} \\
V_{sq}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} \tag{6}
\]

2.1 Diagnostic model
Stator windings short circuits and broken rotor bars are typical electrical faults occurring in induction motors. It is assumed that these electrical faults may be modelled as changes in the stator/rotor electrical parameters about their nominal values \(R_{s0}, L_{s0}, R_{r0}, L_{r0}\). The FDI objective is to detect and isolate any electrical fault when it occurs regardless of load disturbances and mechanical parameter uncertainties. This leads to the following definitions of the fault vector \(f\) and the unknown input vector \(d\):
\[
\begin{bmatrix}
\Delta R_s \\
\Delta L_s \\
\Delta R_r \\
\Delta L_r
\end{bmatrix}
\]
\[
d = \begin{bmatrix}
\delta J \\
\delta v \\
\delta L_t
\end{bmatrix}^T. \tag{8}
\]

Model (5) may be rewritten as:
\[
\dot{x} = A(x) + Bu + D(x)d + F(x,u)f
\]
\[
y = Cx
\]
\[
D(x) = \begin{bmatrix}
\frac{\partial A(x)}{\partial J} & \frac{\partial A(x)}{\partial \delta v} & \frac{\partial A(x)}{\partial T_L}
\end{bmatrix}
\]
\[
F(x,u) = \begin{bmatrix}
\frac{\partial A(x)}{\partial R_s} + B \frac{\partial B}{\partial L_s} u \\
\frac{\partial A(x)}{\partial R_r} + B \frac{\partial B}{\partial L_r} u
\end{bmatrix}
\]

\[\tau_1 = \frac{1}{f^2} (p_{Lm} x_2 x_3 - p_{Lm} x_4 - f_4 x_5 - T_{L_0}), \tau_2 = -x_5/J, \tau_3 = -1/J .\]

\[\text{Eq. (8) represents the on-line residual computation form using only the monitored system input-output variables } (u, y). \text{ The residual evaluation form which shows the residuals dependence on unknown inputs and faults } (d, f) \text{ is obtained by substituting Eq. (7) into Eq. (8):}
\]

\[
r = r(z, x, u, d, f) \tag{9}
\]

The requirements imposed on \((g_R, h_R)\) to achieve robust residual generation are:
(i) \(r(z, x, u, d, 0) = 0\)
(ii) \(r(z, x, u, d, f) \neq 0\)
Condition (i) achieves robustness, so that the residual $r$ is decoupled from unknown inputs $d$, and condition (ii) achieves the sensitivity of $r$ to faults $f$. A fault is said to be detectable if its effect is reflected on the residuals. A fault is said to be isolable if its effect may be discriminated from that of the unknown inputs and the other faults. The fault isolation problem aims at determining the origin of a detected fault. This task may be performed by using adequate residual structures that simplify the isolation decision process.

### 3.2 Unknown input observer with nonlinear decoupling

The transformed IM model (7) is described by the more general nonlinear model which is affine in $(u, f, d)$:

$$
\begin{align*}
\dot{x} &= A(x) + B(x)u + D(x)d + F(x)f \\
y &= C(x)
\end{align*}
$$

Let $n, p, m, n_d, n_f$ denote the dimensions of $x, u, y, d, f$. The synthesis [8], [12] of an unknown input observer for (10) is based on the determination of a nonlinear transformation $z = T(x)$ which defines a new state $z$ decoupled from the unknown input $d$. The decoupling condition [12] is given by:

$$
\frac{\partial T(x)}{\partial x} D(x) = 0
$$

with $\text{rank}(D(x)) = n_d$, and $\text{rank}\left( \frac{\partial T(x)}{\partial x} \right) = \text{rank}\left( \frac{\partial T_1(x)}{\partial x} \ldots \frac{\partial T_{n-n_d}(x)}{\partial x} \right) = n - n_d$

Using Eq. (10) and (11), the decoupled state $z$ is given by:

$$
\begin{align*}
\dot{z} &= \frac{\partial T(x)}{\partial x} \left( A(x) + B(x)u + F(x)f \right) \bigg|_{x = \phi^{-1}(z, y)} \\
y &= C(x) \bigg|_{x = \phi^{-1}(z, y)}
\end{align*}
$$

Let

$$
\phi(x) = \begin{bmatrix} z \\ y' \end{bmatrix} = \begin{bmatrix} T(x) \\ \phi'(y) \end{bmatrix} \bigg|_{y = C(x)}
$$

The inverse function $x = \phi^{-1}(z, y')$ exists under the conditions: $m > n_d$, $\text{rank}\left( \frac{\partial \phi(x)}{\partial x} \right) = n$ and $\lim_{\|x\| \to \infty} \|\phi(x)\| = \infty$. The resulting observer to estimate the decoupled state $z$ is:

$$
\begin{align*}
\dot{x} &= \frac{\partial T(x)}{\partial x} (A(x) + B(x)u + K.R(\dot{x}, y)) \bigg|_{x = \phi^{-1}(\dot{z}, y')} \\
\dot{y} &= C(x) \bigg|_{x = \phi^{-1}(\dot{z}, y')}
\end{align*}
$$

The residual vector is given by:

$$
r = R(\dot{z}, y)
$$

The relation $R(z, y) = 0$ is assumed to be known. The observer gain $K$ which may depend on $(\dot{x}, u)$, is determined, under no fault condition, so that the observer is locally asymptotically stable. The details of the observer synthesis procedure are given in [12]. To ensure that the residual vector is sensitive to faults $f$, the following condition must be satisfied:

$$
\text{rank}\left( \frac{\partial T(x)}{\partial x} F(x) \right) = \text{rank}(F(x))
$$
3.3 Structured residuals for fault isolation
Fault isolation may be achieved by using observer schemes [7], [8], which are structured according to appropriate partitions of the fault vector $f$ into $f^i$ (the set of faults to which the $i^{th}$ observer must be sensitive) and $\tilde{f}^i$ (the remaining set of faults to which the $i^{th}$ observer must not be sensitive), $i=1,...,n_d$. Let the corresponding partition of $F(x)$ into $F^i(x)$ and $\tilde{F}^i(x)$, then the decoupling and sensitivity conditions (11), (15) become:

$$\frac{\partial T(x)}{\partial x} \begin{bmatrix} D(x) & F^i(x) \end{bmatrix} = 0 \quad i=1,...,n_d$$

$$\text{rank} \left( \frac{\partial T(x)}{\partial x} F^i(x) \right) = \text{rank}(F^i(x))$$

(16)

The DOS structure (Dedicated Observer Scheme) is obtained by specifying $f^i$ as the $i^{th}$ fault, and the GOS structure (Generalized Observer Scheme) is reached by defining $f^i$ to be all but the $i^{th}$ fault. In our case, DOS and GOS structures are not achievable for fault isolation in the electrical parameters $(R_s, L_s, R_r, L_r)$ of the IM (conditions (16) not satisfied). However we can restrict the observer scheme to have the following three observer structure: the first observer generates a residual which is robust to the unknown input vector $d$, the second generates a residual which is insensitive to faults in $(R_s, L_s)$, and the third generates a residual which is insensitive to faults in $(R_r, L_r)$.

4. SIMULATION RESULTS
The nominal IM parameters are: $R_s = 10 \ \Omega$, $L_s = 500 \ mH$, $R_r = 5 \ \Omega$, $L_r = 430 \ mH$, $L_m = 430 \ mH$, $J_0 = 0.08 \ kg.m^2$, $f_s = 0.025 \ N.m.s/rad$, $T_L = 1N.m$, $p=2$. Simulation is performed during 12 s with a sampling period of 1 msec. A structured residual generator is implemented according to the design methodology discussed in section 3, so that the robustness and isolation requirements summarized by the following incidence matrix are fulfilled:

$$\Sigma = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(17)

To illustrate the performance of this FDI observer scheme, the following IM fault scenario is simulated: Bias faults of magnitudes $\delta R_s = 50\%$, $\delta R_r = 15\%$, $\delta L_s = 10\%$, $\delta L_r = 5\%$, ($\delta R_s = 50\%$, $\delta R_r = 15\%$) are injected respectively in the time intervals [3,4], [5,6], [7,8], [9,10], [11,12] respectively. A load disturbance $\delta T_L = 20\%$ is injected in the interval [1,2]. Figure 1 displays the time evolution of the corresponding residuals. Figure 2 shows the residuals corresponding to noisy inputs and outputs of the IM. These results are obtained after tuning the dynamics of the observers to give a suitable robustness/detection trade off. It can be seen that only the first residual is robust to the load disturbance. Similar results are obtained for parametric changes in $(J, f_s)$, as expected since they have the same distribution signature. The second residual is not affected by rotor faults, and the third residual is not affected by stator faults. As can be seen the residual vector signature complies with the incidence matrix structure. A simple isolation logic may therefore be implemented in the deterministic case as follows:

$$\begin{align*}
\text{if } r_1 = 0 \text{ and } r_2 \neq 0 \text{ and } r_3 \neq 0 : \text{no fault, } & \forall (J, f_s, T_L) \\
\text{if } r_1 \neq 0 \text{ and } r_2 = 0 \text{ and } r_3 \neq 0 : \text{fault on } Rr/Lr, & \forall (J, f_s, T_L) \\
\text{if } r_1 \neq 0 \text{ and } r_2 \neq 0 \text{ and } r_3 = 0 : \text{fault on } Rs/Ls, & \forall (J, f_s, T_L)
\end{align*}$$
5. CONCLUSION

In this paper, a method of robust fault diagnosis has been applied to the induction machine. A structured residual generation scheme based on unknown input observers with nonlinear decoupling is implemented to achieve detection and isolation of stator and rotor electrical faults independently of external load disturbance and mechanical parameter uncertainties. Simulation results have confirmed the effectiveness of this model-based FDI method. However for faults occurring in one phase only, the hypothesis of a balanced machine no longer holds. This should be handled by more appropriate diagnostic models.

6. REFERENCES